Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 09

B.Tech (CHS) (2018 Batch) (Sem.–1)

MATHEMATICS-I

Subject Code : BTAM-106-18

M.Code : 75368

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions EACH from SECTION - B & C.**

SECTION-A

1. Answer briefly :

a) Are the vectors (1,3,4,2), (3,-5,2,2) and (2,-1,3,2) linearly independent?

b) Find x, y, z and w, given that $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 5 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 6 & x+y \\ z+w & 5 \end{bmatrix}$.

c) Find the adjoint of the matrix $\begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$.

d) Define orthogonal matrix with an example.

e) Define symmetric matrix with an example.

f) If $\vec{r} = a\cos t\,\hat{i} + a\sin t\,\hat{j}$, then find $\nabla\frac{1}{r} = -\frac{\hat{r}}{r^2}$.

g) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then find $\frac{d^2\vec{r}}{dt^2}$.

h) If $\phi(x, y, z) = 3x^2y - y^3z^2$, then find $\nabla\phi$ at the (1, –2, –1).

i) Find constants a, b, c so that $\vec{F} = (x+2y+ax)\hat{i} + (bx-3yz)\hat{j} + (4x+cy+2z)\hat{k}$ is irrotational.

j) If $\vec{F} = 3xy\hat{i} - y^2\hat{j}$, evaluate $\int_C \vec{F}\,.\,\vec{dr}$, where C is the curve in the xy plane $y = 2x^2$ from (0,0) to (1, 2).

SECTION-B

2. a) Find the rank of matrix $\begin{bmatrix} 1 & -3 & 2 \\ 3 & -9 & 6 \\ -2 & 6 & -4 \end{bmatrix}$.

   b) Solve $x - y + z = 2$, $2x + 3y - z = 5$, $x + y - z = 0$ using Gauss elimination method.

3. Examine the consistency of the following system of linear equations and hence, find the solution $4x - y = 12$, $-x + 5y - 2z = 0$, $-2y + 4z = -8$.

4. Express the matrix $\begin{bmatrix} 1 & 3 & -4 \\ -1 & -3 & 4 \\ 2 & 6 & -8 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices.

5. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 1 \end{bmatrix}$.

SECTION-C

6. Show that $grad\, div \vec{V} = Curl\, Curl \vec{V} + \nabla^2 \vec{V}$, where $\vec{V} = V_1 \hat{i} + V_2 \hat{j} + V_3 \hat{k}$.

7. If $\vec{a}$ is a constant vector and $\vec{r}$ is a position vector, show that

   $$Curl\left(\frac{\vec{a} + \vec{r}}{r^3}\right) = -\frac{\vec{a}}{r^3} - \frac{3}{r^3}(\vec{a}.\vec{r})\vec{r} \ .$$

8. Verify Green's theorem in the plane for $\int_C (xy + y^2)\, dx + x^2\, dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

9. Find $\iint_S \vec{F}.\hat{n}\, ds$ where $\vec{F} = (2x + 3z)\hat{i} - (xz + y)\hat{j} + (y^2 + 2z)\hat{k}$ and S is the surface of the sphere having the centre at (3, –1, 2) and radius 3.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.