

Roll No.

Total No. of Pages : 03

Total No. of Questions : 09

B.Tech (Food Technology) (2018 & Onwards) (Sem.-1)

MATHEMATICS-I

Subject Code : BTAM-106-18

M.Code : 75368

Time : 3 Hrs.

Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions EACH from SECTION - B & C.**

SECTION-A

1. Answer briefly :

- Define rank of a matrix.
- For any nonsingular matrix $A = (a_{ij})$ of order n , show that $|Adj(A)| = |A|^{n-1}$
- Determine the values of k for which the system of equations
$$x - ky + z = 0, \quad kx + 3y - kz = 0, \quad 3x + y - z = 0$$
has a nontrivial solution.
- Define orthogonal matrices.
- Is the following matrix diagonalizable? Give reason to your answer.

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

- f) Find the length of the following curve

$$r(t) = a \cos^3 t \mathbf{i} + a \sin^3 t \mathbf{j}, \quad 0 \leq t \leq \pi/2$$

- g) Find gradient of the scalar field $f(x, y, z) = x^2y^2 + xy^2 - z^2$ at $(3, 1, 1)$
- h) Define curl of a vector field.

- i) Find the length of the arc $r(t) = 3 \cos t \mathbf{i} + 3 \sin t \mathbf{j}$, $0 \leq t \leq \pi$.
- j) Evaluate $\int_C x^2 y \, ds$, where C is the curve defined by $x = \cos t$, $y = \sin t$, $0 \leq t \leq \pi/2$.

SECTION-B

2. a) Show that :

$$\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

- b) Solve the following system of equations

$$x - y + 3z = 3, 2x + 3y + z = 2, 3x + 2y + 4z = 5$$

3. a) Use Gauss Jordan method to find the inverse of the matrix

$$\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

- b) For what values of k the following set of vectors form a basis in \mathbb{R}^3 .

$$\{(k, 1 - k, k), (0, 3k - 1, 2), (-k, 1, 0)\}.$$

4. Find all the eigen values and the corresponding eigenvectors of the following matrix.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

5. a) The eigen values of 3×3 matrix A corresponding to the eigenvalues 1, 1, 3 are $[1, 0, -1]^t$, $[0, 1, -1]^t$, $[1, 1, 0]^t$ respectively. Find the matrix A .
- b) Prove that eigen values of a skew-symmetric matrix are zero or purely imaginary.

SECTION-C

6. a) Find directional derivative of the function $f(x, y, z) = xy^2 + 4xyz + z^2$ at a point $(1, 2, 3)$ in the direction of $3i + 4j - 5k$.
- b) If $r = xi + yj + zk$ and $r = |r|$, show that $\text{div}(r/r^3) = 0$.
7. a) For the vector field $v = xyz(yz i + xz j + xy k)$ find a scalar function $f(x, y, z)$ such that $v = \nabla f$.
- b) Find the angle between the surface $z = x^2 + y^2$ and $z = 2x^2 - 3y^2$ at the point $(2, 1, 5)$
8. a) Show that $\int_C (yz - 1)dx + (z + xz + z^2)dy + (y + xy + 2yz)dz$ is independent of the path of integration from $(1, 2, 2)$ to $(2, 3, 4)$. Evaluate the integral.
- b) Evaluate the integral of $v = x^2 i - 2y j + z^2 k$ over the straight line path from $(-1, 2, 3)$ to $(2, 3, 5)$.
9. Find the work done by the force $F = (x^2 - y^2) i + (x^2 + y^2)j$ in moving a particle along a closed path C bounding the region $x^2 + y^2 \leq 16, x^2 + y^2 \geq 4, x \geq 0$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.