

Roll No.

Total No. of Pages : 03

Total No. of Questions : 18

B.Tech. Only for CSE/IT (2018 Batch) (Sem.-1)

**MATHEMATICS-I**

Subject Code : BTAM-104-18

Paper ID : [75362]

Time : 3 Hrs.

Max. Marks : 60

**INSTRUCTIONS TO CANDIDATES :**

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

**SECTION-A**

1. Verify Rolle's theorem for  $f(x) = x(2-x)e^{\frac{3x}{4}}$  in  $(0,2)$
2. Define Beta function and show that it is symmetric.
3. Obtain first three terms of Taylor's series of  $\cos x$  about  $x = \frac{\pi}{4}$ .
4. If  $A + B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$  and  $A - B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$  find  $AB$ .
5. Find rank of the matrix  $\begin{bmatrix} 1 & 6 & -2 \\ 2 & 2 & 1 \\ 3 & 8 & -1 \end{bmatrix}$ .
6. State rank-nullity theorem.
7. Define range of a linear transformation.
8. Define symmetric matrix, also give suitable example.
9. Show that, If zero is an Eigen value of a matrix then it is singular.
10. In an  $n$  dimensional space every set of ' $n + 1$ ' vectors is linearly dependent or independent. Justify your answer.

**SECTION-B**

11. a) Suppose that a function  $f$  is differentiable on  $[0,1]$  and that its derivative is never zero. Using mean value theorem, Show that  $f(0) \neq f(1)$

b) Evaluate the limit  $\lim_{x \rightarrow \frac{\pi}{2}} \left( \frac{1 - \sin x}{\sin x + \cos 2x} \right)$

12. a) Evaluate the integral  $\int_2^{\infty} \frac{2dx}{x^2 - x}$ , if it exists.

b) Find the area of the surface generated by revolving the curve  $y = 2\sqrt{x}$   $1 \leq x \leq 2$  about the x-axis

13. a) Find the inverse of the matrix  $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$  using Gauss Jordan method.

b) Find the rank of the matrix  $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

14. Solve the following system of equations by Cramer's rule

$$2x - 2y + z = 1, x + 2y + 2z = 2, 2x + y - 2z = 7$$

**SECTION-C**

15. a) By giving proper reasoning determine whether S forms a subspace of Vector space  $V$ .

Operations vector addition '+' and scalar multiplications '.' are usual addition and scalar multiplication defined on set of polynomials of degrees less than or equal to 3 ( $P_3$ ) and 3-tuple space ( $V_3$ ).

If (i)  $S = \{p \in P_3 \mid \deg(p) = 3\}, V = P_3$

(ii)  $S = \{(x, y, z) \mid x = 3y\}, V = V_3$

b) Determine whether the following are Linearly dependent or not?

$$x_1 = (1,2,1), x_2 = (2,1,4), x_3 = (1,8,-3)$$

16. a) Let  $V = P_4$ , vector space formed by polynomials of degrees less than or equal to 4 under usual addition and scalar multiplication of polynomials. Find the dimension of subspace  $U$  of  $V$ , where  $U$  is

$$S = \{p \in P_4 \mid p(1) = 0, p'(0) = 0\}$$

- b) Check whether the transformation  $T : V_3 \rightarrow V_2$  defined by  $T(x,y,z) = (x+z, x+y)$  represent a Linear transformation or not?

17. Find the Eigen values and Eigen vectors for the matrix.

$$\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

18. a) If  $A$  is an orthogonal matrix prove that  $|A| = \pm 1$

- b) Define similar matrices and prove that similar matrices have same eigen values.