Roll No. Total No. of Pages : 2

Total No. of Questions : 07

M.Sc. Mathematics (2017 Batch) (Sem.–1)

# ALGEBRA-I

Subject Code : MSM-101

Paper ID : [74720]

Time : 3 Hrs. Max. Marks : 80

**INSTRUCTIONS TO CANDIDATES :**

1. SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.
2. SECTION - B & C. have THREE questions in each section carrying SIXTEEN marks each.
3. Select atleast TWO questions from SECTION - B & C EACH.

## SECTION-A

1. **Answer briefly :**

a) Write all abelian groups of order 108.

b) State Sylow's second theorem.

c) Let R be a ring and $a \in R$. Prove that $I = \{x \varepsilon R/ax = 0\}$ is a right ideal of R.

d) Prove that any p-Sylow subgroup of a group G of order 33 is a normal subgroup of G.

e) Show that elements in the same class of a group must have the same order.

f) Write the composition series for a cyclic group of order 50.

g) Write the composition series for the group $A_4$.

h) Prove that group of order 55 is not simple.

## SECTION-B

2 a) Prove that $A_n$, $n>4$, is the only nontrivial normal subgroup of $S_n$.

b) Let G be a group of order 2m where m is odd. Prove that G contains a normal subgroup of order m.

3 a) Prove that if an abelian group has a composition series, then G is a finite group.

b) State and prove Cayley's theorem.

4 a) Prove that any two finite sub normal series for a group G have isomorphic refinements.

b) Write the composition series for the symmetric group $S_4$.

**SECTION-C**

5 a) Prove that a divison ring is a simple ring.

b) Let G be a finite group such that $x^2 = e$ for all $x \varepsilon G$. Prove that G is the direct product of a finite number of cyclic groups of order 2.

6 a) State and prove sylow's third theorem.

b) Prove that there are only two non-abelian groups of order 8.

7 a) Prove that in an integral domain every prime element is an irreducible element. The converse may not be true.

b) Let R be a Boolean ring. Then each prime ideal $P \neq R$ is maximal.