

Roll No.

--	--	--	--	--	--	--	--	--	--	--	--

Total No. of Pages : 02

Total No. of Questions : 07

M.Sc. Mathematics (2017 Batch) (Sem.-1)

COMPLEX ANALYSIS

Subject Code : MSM-103

Paper ID : [74722]

Time : 3 Hrs.

Max. Marks : 80

INSTRUCTION TO CANDIDATES :

1. **SECTION-A** is **COMPULSORY** consisting of **EIGHT** questions carrying **TWO** marks each.
2. **SECTION - B & C.** have **THREE** questions in each section carrying **SIXTEEN** marks each.
3. **Select atleast TWO questions from SECTION - B & C EACH.**

SECTION-A

1) Answer briefly :

- a) Evaluate $(1-i)^4$.
- b) Prove that $\operatorname{Re}(iz) = -\operatorname{Im} Z$.
- c) Check whether the function $f(z) = \operatorname{Im}(z)$ has a derivative at any point.
- d) Find the singular points of $\frac{z^3 + i}{z^2 - 3z + 2}$.
- e) Evaluate $\operatorname{Log}(-ei)$.
- f) Write MitagLefler's inequality of complex integration and explain it.
- g) State Rouche's theorem.
- h) Discuss the types of singularities.

SECTION-B

- 2) a) Find all roots of $(2i)^{1/2}$ and exhibit them geometrically.
b) Show that the function $f(z) = \bar{z}$ is nowhere differentiable.
- 3) a) Show that the function $f(z) = \frac{\overline{(z)}^2}{z}$ if $z \neq 0$ and 0 if $z = 0$ is not differentiable at $z = 0$ but the CR equations are satisfied.
b) Use the polar form to show that $-(1+i)^7 = -8(1+i)$
- 4) a) Determine accumulation points of $z_n = \left(\frac{1}{n}\right)i^n (n=1, 2, \dots)$.
b) Establish Cauchy riemann equations.

SECTION-C

- 5) a) Show that $u(x,y) = \sinh x \sin y$ is harmonic in some domain and find its harmonic conjugate of it.
b) Show that when $n = 0, \pm 1, \pm 2, \dots$
$$(1+i)^i = \exp\left(-\frac{\pi}{4} + 2n\pi\right) \exp\left[\frac{i}{2} \log 2\right]$$
- 6) a) State and prove mitag lefler's inequality.
b) Determine the domain of analyticity of the function f and apply Cauchy Goursat theorem to show that $\int \frac{z^2}{z-3} dz = 0, |z|=1$
- 7) a) State and prove maximum modulus principle.
b) Establish Schwarz lemma.