

Roll No.

Total No. of Pages : 02

Total No. of Questions : 07

**M.Sc Mathematics (2017 Batch) (Sem.-1)**  
**ORDINARY DIFFERENTIAL EQUATIONS AND SPECIAL**  
**FUNCTIONS**

Subject Code : MSM-104

Paper ID : [74723]

Time : 3 Hrs.

Max. Marks : 80

**INSTRUCTION TO CANDIDATES :**

1. SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.
2. SECTION - B & C. have THREE questions in each section carrying SIXTEEN marks each.
3. Select atleast TWO questions from SECTION - B & C EACH.

**SECTION-A**

**1. Answer briefly :**

- a. When is a singular point said to be regular?
- b. State Bessel's function of order n.
- c. Show that  $\int_{-1}^1 x P_n(x) P_{n-1}(x) dx = \frac{2x}{4n^2 - 1}$ .
- d. Give the generating function for Laguerre's Polynomial.
- e. Formulate the differential equation of all circles of radius a.
- f. Find general solution of  $(2xy + x^2)y' = 3y^2 + 2xy$ .
- g. Define the Ber and Bei functions.
- h. State Picard's fundamental existence and uniqueness theorem.

### SECTION-B

2. State and prove Sturm's separation theorem.
3. Solve  $x^2 \left(\frac{dy}{dx}\right)^2 + xy \frac{dy}{dx} - 6y^2 = 0$
4. Express  $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$  in terms of Legendre polynomials.

### SECTION-C

5. a) Solve in series  $xy'' + 2y' + xy = 0$   
b) Express  $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$  in terms of Legendre Polynomials.
6. Prove that  $(2n + 1)(1 + x^2) P_n'(x) = n(n + 1) [P_{n+1}(x) - P_{n-1}(x)]$
7. Define Hermite Polynomials. Hence establish the expression for the generating polynomial.