Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 07

M.Sc. (Mathematics) (2017 Batch) (Sem.–1)

REAL ANALYSIS – I

Subject Code : MSM-102

M.Code : 74721

Time : 3 Hrs. **Max. Marks : 80**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.**
2. **SECTION - B & C have THREE questions in each section carrying SIXTEEN marks each.**
3. **Select atleast TWO questions from SECTION - B & C EACH.**

SECTION-A

1. a) Show that the series $\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\frac{8x^7}{1+x^8}+....$ is uniform convergent in $x\in\left[-\frac{1}{2},\frac{1}{2}\right]$.

b) Whether $d:\ \mathbb{R}\times\mathbb{R}\to\mathbb{R}$ defined by $d(x, y) = |x| + |y|$ is a metric on $\mathbb{R}$?

c) Evaluate $\int_0^4 xd([x]-x)$

d) Show that arbitrary union of closed sets may or may not be closed.

e) If X is a metric space and $E \subset X$, then interior of set E is open.

f) If $x, y \in \mathbb{R}$ and $x > 0$, then there is a positive integer n such that $nx > y$.

g) Show that the series $\sum\frac{\sin nx}{n^p}$ is uniformly and absolutely convergent for all real value of x and $p > 1$.

h) Is $\mathbb{R}-[a, b]$ dense in $\mathbb{R}$, where a and b are two distinct real numbers.

SECTION-B

2. a) Show that the Thomae's function is discontinuous at rationals and continuous at irrationals.

b) Show that the space $\mathbb{R}$ with usual metric is complete. (8+8)

3. If A and B are two subsets of a metric space (X, d), then :

a) If A $\subseteq$ B, then $\bar{A} \subseteq \bar{B}$.

b) $A = \bar{A} \Leftrightarrow$ A is closed.

c) $\overline{(A \cup B)} = \bar{A} \cup \bar{B}$

d) $\overline{(A \cap B)} \subseteq \bar{A} \cap \bar{B}$ (4 × 4 = 16)

4. If $f \in$ R (α) on $[a, b]$ iff for every ε > 0 there exists a partition P such that U (P, f, α) – $L(P, f, \alpha) < \varepsilon$. (16)

SECTION-C

5. a) If f is continuous and α is monotonically increasing on $[a, b]$, then there exists a number η ∈ $[a, b]$ such that $\int_a^b f d\alpha = f(\eta)[\alpha(b) - \alpha(a)]$.

b) If F is closed and E is compact subset of a compact metric space, then F ∩ K is compact. (8 + 8)

6. a) Show that finite intersection of open sets is open, that is, if G_1, …. G_n are open sets then $\bigcap_i^n G_i$ is open.

b) The field $\mathbb{Q}$ does not have least upper bound property. (8 + 8)

7. a) Show that $f_n(x) = \dfrac{nx}{1+n^2x^2}$, for all $x \in \mathbb{R}$ is point wise convergence but not uniform convergence.

b) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E and suppose that $\lim\limits_{t\to x} f_n(t) = A_n$, n = 1, 2, 3, ….. Then $\{A_n\}$ converges and $\lim\limits_{t\to x} f(t) = \lim_{n\to\infty} A_n$. (6+10)

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.