http://www.punjabpapers.com

Roll No. | | | | | | | | | | | | **Total No. of Pages : 02**

Total No. of Questions : 07

M.Sc. Mathematics (2018 Batch) (Sem.-1)

REAL ANALYSIS-I

Subject Code : MSM-102-18

M.Code : 75130

Time : 3 Hrs. **Max. Marks : 70**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.**
2. **SECTION - B & C have THREE questions each.**
3. **Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.**
4. **Select atleast TWO questions from SECTION - B & C each.**

SECTION-A

1. Answer the following :

a) Consider $(\mathbb{R}, d)$ a metric space. Is $\bigcup_{n=1}^{\infty}(-n, n)$ cover of $\mathbb{R}$?

b) Let $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined as :

$$d(x, y)=\begin{cases}|x|+|y|; x\neq y\\ 0; x=y\end{cases}$$

Is d metric over $\mathbb{R}$? Verify your result.

c) Show that $\int_0^3 x d([x]-x)=\frac{3}{2}$

d) Show that $\int_0^1\left(\sum_{i=1}^{n}\frac{x^n}{n^2}\right)=\sum_{n=1}^{\infty}\frac{1}{n^2(n+1)}$.

e) Show that the series $\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\frac{8x^7}{1+x^8}+....$ is uniform convergent in $x\in\left[-\frac{1}{2},\frac{1}{2}\right]$.

http://www.punjabpapers.com

http://www.punjabpapers.com

SECTION-B

2. a) Every bounded real sequence has a convergent subsequence. (10)

   b) Suppose (X, d_X), (Y, d_Y), (Z, d_Z) are metric spaces. If $f : X \to Y$ and $g : Y \to Z$ are continuous then the composition $h = g o f : X \to Z$ is continuous. (5)

3. a) Let (X, d) be a complete metric space and $A \subseteq X$. Then A is complete iff A is closed. (8)

   b) Let A and B be countable subsets of a metric space (X, d). Then :

   i) $A \cup B$ is countable

   ii) $A \times B$ is countable. (7)

4. a) Compact subsets of metric spaces are closed. (7)

   b) Closed subsets of a compact metric space are compact. (8)

SECTION-C

5. a) If f is monotonic on $[a, b]$ and α is continuous on $[a, b]$, then $f \in R(\alpha)$. (10)

   b) Show that $f_n(x) = \dfrac{nx}{1+n^2x^2}$, for all $x \in \mathbb{R}$ is pointwise convergence but not uniform convergence. (5)

6. Assume that α increases monotonically and $\alpha' \in R$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then $f \in R(\alpha)$ iff $f\alpha' \in R$ (15)

   In that case $\int_a^b f \, d\alpha = \int_a^b f(x)\, \alpha'(x)\, dx$.

7. Suppose that $\{f_n\}$ is a sequence of functions, differentiable on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a, b]$

   If $\{f_n'\}$ converges uniformly on $[a, b]$ then $\{f_n\}$ converges uniformly on $[a, b]$ to a function f, and $f'(x) = \lim_{n\to\infty} f_n'(x), a \le x \le b$. (15)

http://www.punjabpapers.com

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.

http://www.punjabpapers.com