Roll No.

Total No. of Pages: 03

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M.Sc. Mathematics (2017 Batch) EL-I (Sem.-3)

CODING THEORY

Subject Code: MSM-501 M.Code: 75385

Time: 3 Hrs. Max. Marks: 80

INSTRUCTIONS TO CANDIDATES:

- 1. SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.
- 2. SECTION B & C have THREE questions in each section carrying SIXTEEN marks each.
- 3. Select atleast TWO questions from SECTION B & C EACH.

SECTION-A

l. Answer briefly:

- a) Show that code C can correct upto t errors in any codewords if $d(C) \ge 2t + 1$.
- b) Let C be the subspace of V(4, 3), having generating set $\{(0, 1, 2, 1), (1, 0, 2, 2), (1, 2, 0, 1)\}$. Find basis of C. What is dim C?
- c) Write down the parity check matrix for *Ham* (4, 2).
- d) Find the (multiplicative) order of $x \mod (x^3 + x + 1)$ with coefficients in $\mathbb{Z}/2$.
- e) Prove that two vectors u and v are in same coset if and only if they have the same syndrome.
- f) Find primitive element for GF(7).
- g) Show that binary even weight code is cyclic.
- h) Show that an [n, n-r, d] code satisfies $d \le r+1$.

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SECTION-B

- 2. a) Let C_1 be a binary (n, M_1, d_1) code and C_1 be a binary (n, M_2, d_2) code. Consider $C = \{u|u+v, u \in C_1, v \in C_2\}$. Then show that C is $(2n, M_1M_2, d)$ code where $d = \min\{2d_1, d_2\}$.
 - b) Prove that in binary linear code either all the code words have even weight or exactly half have an even and half have odd weight.
- 3. a) Prove that the binary Hamming code Ham (r, 2) for $r \ge 2$
 - i) Is $(2^r 1, 2^r 1 r)$ code
 - ii) Has minimum distance 3
 - iii) Is a perfect code
 - b) Let C be binary linear code with generator matrix

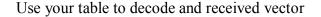
$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Find a generator matrix for C in standard form.

- 4. a) Suppose $[I_K|A]$ is a standard form generator matrix linear code C. Show that any permutation of the rows of A gives generator matrix for a code which is equivalent to C.
 - b) Construct a syndrome look-up table for the perfect binary [7, 4, 3] code which has generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

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- i) 0000011
- ii) 1000011
- iii) 1111111
- iv) 1100110
- v) 1010101

SECTION-C

- 5. Determine the dimension and minimum distance of the BCH code of length 48 constructed with designed distance 9 using the field extension GF (7²) of the finite field GF (7).
- 6. a) An irreducible polynomial P of degree N in $F_q[x]$ is primitive if and only if P divides the $(q^N 1)^{th}$ cyclotomic polynomial in $F_q[x]$.
 - b) Suppose C is cyclic code with generator polynomial $g(x) = g_0 + g_1(x) + \dots + g_r(x')$. Then prove that dim (c) = n r, also find the generator matrix of the code C.
- 7. Suppose C is an [n, n-r] code with parity check matrix $H = [A^T]I_r$. Then prove that C is an MDS code if and only if every square sub matrix of A is non singular.

NOTE: Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.

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