Roll No. Total No. of Pages : 03

Total No. of Questions : 07

M.Sc. Mathematics (2017 Batch) EL-I (Sem.–3)

CODING THEORY

Subject Code : MSM-501

M.Code : 75385

Time : 3 Hrs. Max. Marks : 80

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.
2. SECTION - B & C have THREE questions in each section carrying SIXTEEN marks each.
3. Select atleast TWO questions from SECTION - B & C EACH.

SECTION-A

l. **Answer briefly :**

a) Show that code C can correct upto t errors in any codewords if d (C) $\geq 2t + 1$.

b) Let C be the subspace of V(4, 3), having generating set {(0, 1, 2, 1), (1, 0, 2, 2), (1, 2, 0, 1)}. Find basis of C. What is dim C?

c) Write down the parity check matrix for *Ham* (4, 2).

d) Find the (multiplicative) order of $x \bmod (x^3 + x + 1)$ with coefficients in Z/2.

e) Prove that two vectors u and v are in same coset if and only if they have the same syndrome.

f) Find primitive element for GF (7).

g) Show that binary even weight code is cyclic.

h) Show that an $[n, n - r, d]$ code satisfies $d \leq r + 1$.

SECTION-B

2. a) Let C_1 be a binary (n, M_1, d_1) code and C_1 be a binary (n, M_2, d_2) code. Consider $C = \{u|u + v, u \in C_1, v \in C_2\}$. Then show that C is $(2n, M_1M_2, d)$ code where $d = \min \{2d_1, d_2\}$. 12

b) Prove that in binary linear code either all the code words have even weight or exactly half have an even and half have odd weight. 4

3. a) Prove that the binary Hamming code Ham $(r, 2)$ for $r \geq 2$ 10

i) Is $(2^r - 1, 2^r - 1 - r)$ code

ii) Has minimum distance 3

iii) Is a perfect code

b) Let C be binary linear code with generator matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Find a generator matrix for C in standard form. 6

4. a) Suppose $[I_K|A]$ is a standard form generator matrix linear code C. Show that any permutation of the rows of A gives generator matrix for a code which is equivalent to C. 6

b) Construct a syndrome look-up table for the perfect binary [7, 4, 3] code which has generator matrix 10

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Use your table to decode and received vector

i) 0000011

ii) 1000011

iii) 1111111

iv) 1100110

v) 1010101

SECTION-C

5. Determine the dimension and minimum distance of the BCH code of length 48 constructed with designed distance 9 using the field extension GF (7^2) of the finite field GF (7). 16

6. a) An irreducible polynomial P of degree N in $F_q[x]$ is primitive if and only if P divides the $(q^N - 1)^{th}$ cyclotomic polynomial in $F_q[x]$. 10

   b) Suppose C is cyclic code with generator polynomial $g(x) = g_0 + g_1(x) \ldots \ldots + g_r(x^r)$. Then prove that dim $(c) = n - r$, also find the generator matrix of the code C. 6

7. Suppose C is an $[n, n - r]$ code with parity check matrix $H = [A^T]I_r]$. Then prove that C is an MDS code if and only if every square sub matrix of A is non singular. 16

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.