Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 07

M.Sc. Mathematics (2017 Batch) (Sem.-3)

TOPOLOGY

Subject Code : MSM-301

M.Code : 75381

Time : 3 Hrs. **Max. Marks : 80**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.**
2. **SECTION - B & C have THREE questions in each section carrying SIXTEEN marks each.**
3. **Select atleast TWO questions from SECTION - B & C EACH.**

SECTION-A

Q1. Answer briefly :

a. Define bases for a topological space.

b. Let (X, τ) be a topological space, where $X = [1, 10]$ and τ is the indiscrete topology. What will be the derived set of $A = \{2, 3\}$.

c. Define open mapping in a topological space. Give an example.

d. Consider the usual topology on the set of natural numbers |N. Check whether this topological space is connected or not? Justify your claim.

e. Let (X, τ), where $X = \{1, 2, 3\}$, $\tau = \{\theta, X, \{1, 3\}, \{2\}\}$, be a topological space. Check whether this topological space is Hausdorff or not?

f. Show that the property of a topological space being T_0 is hereditary property.

g. Define a completely normal topological space. Give an example.

h. State Tietze Extension theorem. $(8 \times 2 = 16)$

SECTION-B

Q2. Let (X,τ) be any topological space and $A \subset X$. Let B (A) denote the boundary of A. Then

a) Show that : (8)

i) $x \in B(A)$ if and only if $x \in \overline{A} \cap (\overline{X - a})$

ii) $x \in A^{\circ}$ iff $x \in A - B(A)$.

b) Show that A is closed iff it contains all its limit points. (8)

Q3. Show that every T_1 and regular topological space with a countable base is normal. (16)

Q4. a) Prove that every compact subset A of a Hausdorff space X is closed. (8)

b) Let (Y, τ_Y) be a subspace of a topological space (X,τ) and let A and B be two subsets of Y. Then show that A and B are τ-separated if they are τ_Y-separated. (8)

SECTION-C

Q5. a) Let B* be a non-empty collection of subsets of a non-empty set X. Then show that B* forms a sub-base for a unique topology τ for X. (8)

b) Let X and Y be two topological spaces. Then show that a mapping $f : X \to Y$ is continuous if and only if the inverse image of every member of a base for Y is open in X. (8)

Q6. Show that every second countable topological space is first countable, but converse is not true. (16)

Q7. State and prove Urysohn's lemma. (16)

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.