Roll No. $\square$
Total No. of Questions: 08

## Ph.D in Faculty of Applied Science (Mathematical Sciences)

ADVANCED NUMBER THEORY
M.Code: 77358

Time : 3 Hrs.

## INSTRUCTIONS TO CANDIDATES :

1. Attempt any FIVE questions out of EIGHT question.
2. Each question carry TWENTY marks.
3. a) Prove that the number of partitions of $n$ into $m$ parts is the same as the number of $n$ having largest part $m$. Similarly, the number of partitions of $n$ into at most $m$ parts is equal to $p_{m}(n)$, the number of partitions of $n$ into parts less than or equal to $m$.
b) Prove that $p^{d}(n)=p^{o}(n)$ for $n \geq 1$.
4. a) State Sylvester's theorem, State and prove q-binomial theorem.
b) Establish Jacobi's triple product identity.
5. a) What is Restricted partitions? Derive Rogers Lemma.
b) What is Gaussian polynomials, prove q-Gauss theorem?
6. State $q$-Saalschutz's theorem, derive finite version of $q$-Saalschutz's theorem. State Biley's lemma(weak version).
7. a) State and prove Schur's theorem.
b) Derive Gollnitz-Gordon identities.
8. State Watson's q-analogue of Whipple's theorem, and derive Rogers-Ramanujan identities as its application.
9. What is $n$-colour partitions, explain Rank and Crank of a partition with two examples?
10. What is Restricted $n$-colour partitions, explain conjugate and self-conjugate $n$-partition with three examples?

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.

