Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 03**

Total No. of Questions : 18

B.Sc. (Honours) Mathematics (PIT) (2019 Batch) (Sem.–1)

CALCULAS-I

Subject Code : UC-BSHM-101-19

M.Code : 77312

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. Show that a set S is bounded iff there exists a real number $M > 0$ such that set $|x| < M \ \forall \ x \in S$.

2. Give an example of two functions f and g such that fog is the zero function even when f and g are not zero functions.

3. Differentiate $\sqrt{x} \cos ech\sqrt{x}, x > 0$.

4. Differentiate $\sin^{-1}(3x - 4x^3)$, with respect to x if $-\frac{1}{2} < x < \frac{1}{2}$.

5. If $f(x) = 2x^{\frac{2}{3}}$, $a = -1$, $b = 1$, show that there is no real number c which satisfies the Lagrange's Mean Value theorem.

6. Evaluate $\lim\limits_{x \to \frac{\pi}{2}} \frac{\log \sin x}{1 - \sin x}$.

7. Find the horizontal and vertical asymptotes of the curve $x^2 y^2 - a^2 (x^2 + y^2) = 0$.

8. Find the nth derivative of $\frac{x}{1+3x+2x^2}$.

9. By using definition, prove that $f(x)=\begin{cases} x^2\cos\frac{1}{x} & : \quad x\neq 0 \\ 0 & : \quad x=0 \end{cases}$ is continuous at $x = 0$.

10. Show that every point at which the sine curve $y = c\sin\frac{x}{a}$ meets the axis of x is a point of inflexion of the curve.

SECTION-B

11. (a) State and prove Archimedean property of real numbers.

(b) If $\lim_{x\to a} f(x)$ exists, then show that it is unique.

12. (a) Prove that $f(x) = x^2$ is continuous in R but is not uniformly continuous on R.

(b) Find all the asymptotes of the following curve :

$$x^3 + x^2y + xy^2 + y^3 + 2x^2 + 3xy - 4y^2 + 7x + 2y = 0$$

13. (a) If $f(x)$, $g(x)$ and $h(x)$ are three polynomials of degree 2, then prove that $\phi(x)=\begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$ is a constant polynomial.

(b) If $x\sqrt{1-y}+y\sqrt{1+x}=0$ and $x \neq y$, prove that $\frac{dy}{dx}=-\frac{1}{(x+1)^2}$.

14. (a) If $y = e^{m\sin^{-1}x}$, show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-m^2y=0$.

(b) Show that $\frac{d}{dx}$ (tan $h(\log x)) = \frac{4x}{(x^2+1)^2}$.

SECTION-C

15. (a) Find the points of inflexion, interval of rising and falling, concave upwards and concave downwards for the curve $y = \frac{x^2+1}{x^2-1}$.

    (b) Use Mean Value theorem to find the approximate value of $\sqrt{66}$.

16. (a) Find the values of p and q, so that the $\lim_{x \to 0} \frac{x(1+p\cos x) - q\sin x}{x^3}$ exists and its equal to 1.

    (b) Use Maclaurin's Theorem (with Lagrange's form of remainder) to expand $\cos x$.

17. (a) Find the nth derivative of $\sin^2 x \cos 4x$.

    (b) If $y = \sin(m \sin^{-1} x)$, show that $\lim_{x \to 0} \frac{y_{n+2}}{y_n} = n^2 - m^2$.

18. (a) Write out the first three non-zero terms in Taylor's formula for the function $f(x) = \sin^2 x, x_0 = 0$.

    (b) Use Cauchy's Mean Value theorem to evaluate $\lim_{x \to 1} \frac{\cos \frac{\pi x}{2}}{\log \frac{1}{x}}$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.