Roll No. | | | | | | | | | | | | Total No. of Pages : 02

Total No. of Questions : 18

B.Sc. (Non Medical) (2018 & Onwards) (Sem.–1)

DIFFERENTIAL CALCULUS

Subject Code : BSNM-105-18

M.Code : 75746

Time : 3 Hrs. **Max. Marks : 50**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying ONE mark each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

Answer briefly :

1. Show that $\lim_{n\to\infty}\left(\frac{2}{1}.\frac{3}{2}.\frac{4}{3}....\frac{n}{n-1}\right)^{\frac{1}{n}} = 1$.

2. Show that the series $\sqrt{\frac{1}{4}}+\sqrt{\frac{2}{6}}+\sqrt{\frac{3}{8}}+....+\sqrt{\frac{n}{2(n+1)}}+.....$ does not converge.

3. Prove that the limit $\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^4y^4}{(x^2+y^4)^3}$, does not exist.

4. Examine the derivability of $|x|$ at $x = 0$.

5. State Rolle's Theorem.

6. Find $\frac{dy}{dx}$ when $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$.

7. Let $f : R^2 \to R$ be defined by $f(x,y) = \begin{cases} 1, & if\ (x,y) \neq (0,0) \\ 0, & if\ (x,y) = (0,0) \end{cases}$. Prove that the first order partial derivatives of f does not exist at (0, 0).

8. Show that the function $f(x) = x^2$ is uniformly continuous on [–1, 1].

9. Find $\lim_{x\to 0}\frac{x-\tan x}{x^3}$.

10. If $a_n = \frac{(-1)^n}{n^2}$, n is a natural number then show that $\underline{Lim}\, a_n = 0 = \overline{Lim}\, a_n$.

SECTION-B

11. If $a_n \to l$ then show that $x_n = \frac{a_1 + a_2 + a_3 + ... + a_n}{n} \to l$.

12. If $x^x. y^y. z^z = c$ then show that at $x = y = z, \frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$.

13. The function $f(x)$ is defined as $f(x) = \begin{cases} ax^2 + b, & if\ x > 2 \\ 2, & if\ x = 2 \\ 2ax - b, & if\ x < 2 \end{cases}$. If $f(x)$ is continuous everywhere then find the values of a and b.

14. If $x = \frac{1-t^2}{1+t^2}, y = \frac{2t}{1+t^2}$ then prove that $\frac{dy}{dx} + \frac{x}{y} = 0$.

15. If z be a homogeneous function of x and y of order n then show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$ for all x and y of the domain of the function.

SECTION-C

16. a) Prove that the sequence $\{a_n\}$, where $a_n = 8 + \frac{1}{n^3}$ is a Cauchy sequence and find its limit.

    b) Discuss the convergence or divergence of the series $\sum \frac{\sqrt{n^2 + n + 1} - \sqrt{n^2 - n + 1}}{n}$.

17. a) Show that the function $f(x, y) = \sin x + \sin y$ is differentiable at every point of R^2.

    b) Show that the functions $u = x^2 + y^2 + z^2 - 2zx$, $v = x + y - z$, $w = x - y - z$ are not independent of one another. Also find the relation between them.

18. a) State the prove Taylor's Theorem.

    b) Evaluate $\lim_{x\to\infty}(\sqrt{x^2 + x + 1} - x) \neq \lim_{x\to\infty}(\sqrt{x^2 + 1} - x)$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.