Roll No. $\square\square\square\square\square\square\square\square\square\square\square\square$ **Total No. of Pages : 03**

Total No. of Questions : 18

Bachelor of Science - Honours (Mathematics) (Sem.–3)

CALCULUS-III

Subject Code : UC-BSHM-301-19

M.Code : 78496

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. Define Homogeneous function. Give an example.

2. If $J = \frac{(u,v)}{(x,y)}$ and $J' = \frac{(x,y)}{(u,v)}$ then prove that $JJ' = 1$.

3. The period of simple pendulum is T = $2n\sqrt{(1/g)}$, find the maximum error in r due to the possible error upto 1% in l and 2.5% in g.

4. A lamina is bounded by the curves $y = x^2 - 3x$ and $y = 2x$. If density at any point is given by λxy. Find by double integration, the mass of the lamina.

5. If the density of any point of the solid contant of the ellipsoid $\frac{x2}{a2}+\frac{y^2}{b^2}+\frac{2^2}{C^2}=1$ is given by $\rho = xyz$. Find the coordinates of the centre of gravity of the solid.

6. Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.

7. If $u = e^{xyz}$, find the value of $\frac{\partial^2 u}{\partial x \partial y \partial z}$

8. Evaluate $\int_0^x \int_0^x \int_0^{x+y} e^{x+y+z}\, dxdydx$

9. Transform $\iint dxdy$ into polar coordinates.

10. Explain graphically the meaning of partial derivative.

SECTION-B

11. i) Investigate the continuity of the function

$$f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2}, & \text{if } (x,y)\neq(0,0)\\ 0, & \text{if } (x,y)=(0,0)\end{cases}$$

at the origin.

ii) Evaluate the limit $\lim\limits_{\substack{x\to 1\\ y\to 1}}\dfrac{x(y-1)}{y(x-1)}$ display style $\dfrac{x(y-1)}{y(x-1)}$.

12. i) State Euler's theorem on homogeneous functions and hence prove that if $u = xln^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$, then $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=3\tan(u)$.

ii) If f = $\theta = t^n e^{-r^2/4t}$, what value of n will make $\dfrac{1}{r^2}\dfrac{\theta\left(r^2\dfrac{\partial u}{\partial r}\right)}{\partial r}=\dfrac{\partial\theta}{\partial t}$.

13. i) If $u = x^2 + y^2 + z^2$ and $x = e^{2t}$, $y = e^{2t}\cos 3t$, $z = e^{3t}\sin 3t$. Find $\dfrac{du}{dt}$ as a total derivatives and verify the result by direct substitution.

ii) If $u = x\log(xy)$, where $x^2 + y^3 + 3xy = 1$. Find $\dfrac{du}{dx}$.

14. i) If $u=\dfrac{x+y}{1-xy}$ and $v = \tan^{-1}x + \tan^{-1}y$. Find $\dfrac{d(u,v)}{d(x,y)}$ are u and v functionally related if so, find the relationship.

ii) If $u = x^2 - y^2$, $v = 2xy$ and $x = r\cos\theta$, $y = r\sin\theta$, then find $\dfrac{d(u,v)}{d(r,\theta)}$.

SECTION-C

15. Expand $f(x, y) = \tan - 1(y/x)$ in powers of $(x - 1)$ and $(y - 1)$ up to third degree terms. Also, compute $f(1.1, 0.0)$ approximately.

16. i) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

    ii) The temperature T at any point (x, y, z) in space is $T = 400xyz^x$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + x^2 = 1$.

17. i) Change the order of integration in $r = \int_0^1 \int_{x^2}^{2-x} xydxdy$ and hence evaluate.

    ii) Evaluate $\iint_R (x + y)^2 dxdy$, where R is the parallelogram in the xy-plane with vertices (1, 0), (3, 1), (2, 2), (0, 1) using the transformation $u = x + y$ and $v = x - 2y$.

18. i) Find the volume bounded by the xy-plane, the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$.

    ii) Find by triple integration, the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.