Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 07

B.Sc.(CS) (2013 Batch) (Sem.–6)

LINEAR ALGEBRA

Subject Code : BCS-602

Paper ID : [72782]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains SIX questions carrying TEN marks each and students has to attempt any FOUR questions.**

SECTION-A

Q1 Answer the followings in short :

a) Define Division ring and Field with examples.

b) Show that $\{1, \sqrt{2}\}$ is linearly independent in R over Q.

c) Under what condition on scalar τ do the vectors (1, 1,1), $(1, \tau, \tau^2)$ and $(1, - \tau, \tau^2)$ forms a basis of C^3 ?

d) Define Quotient Space and its dimension.

e) Any n+1 members of vector space V of dimension n are Linearly Dependent. Prove it.

f) State Sylvester Law of Nullity.

g) Check whether a mapping T: $R^3 \rightarrow R$ defined by $T(x, y, z) = x^2 + y^2 + z^2$ is a linear transformation?

h) Determine the complement of the subspace of V generated by {(1, 1, 0), (0, 1, 0)}.

i) Define the annihilator of linear transformation of vector space over field,

j) Find the co-ordinate vectors if v in R^3 relative to the basis {(1, 1, 1), (l, l, 0), (1, 0, 0)}.

SECTION-B

Q2 If U and W are subspaces of a finite dimensional vector space over F then prove that

$\dim(U+W) = \dim(U) + \dim(W) - \dim(U \cap W)$

Q3 a) Let V be a vector space over an infinite field. Prove that V cannot be written as set theoretic union of a finite number of proper subspaces z.

b) If x, y and z are vectors in vector space over F such that $x + y + z=0$, then show that x and y span the same subspace as y and z.

Q4 Let V be a finitely generated vector space over a field. Prove that V has a finite basis and any two bases of V have same number of vectors.

Q5 Let U and V be two subspaces of a vector space W. Show that $(U + V) / U \cong V /(U \cap V)$.

Q6 a) Show that a linear transformation $T: V \rightarrow W$ is non-singular iff T carries each linearly independent subspace of V onto linearly independent subspace of W.

b) Let T be a linear operator on V and $\text{Rank}(T^2) = \text{Rank}(T)$. Then show that the $\text{Range}(T) \cap \text{Ker}(T) = (0)$.

Q7 Show that there is a one-one correspondence between direct decomposition of a vector space and finite sets of supplementary orthogonal projections on that vector space.