Roll No.

Total No. of Pages : 02

Total No. of Questions : 07

B.Sc. (Computer Science) (2013 & Onwards) (Sem.–6)

REAL ANALYSIS

Subject Code : BCS-601

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains SIX questions carrying TEN marks each and students have to attempt any FOUR questions.**

SECTION–A

Q**1. Write briefly :**

(a) Does point wise convergence imply uniform convergence? Justify.

(b) Find radius of convergence of power series :

$$\sum\left(1+\frac{1}{n}\right)^{n^2} x^n$$

(c) Prove that the series obtained by differentiating a power series term by term has the same radius of convergence as the original series.

(d) Verify that product of two uniformly convergent sequences need not be convergent.

(e) Discuss that properties of power series.

(f) Prove that Re(iz) = – im z

(g) Prove that $\sqrt{2}\,|z| \geq |\mathrm{Re}\,z| + |im\,z|$.

(h) Find the roots of $(-1)^{1/3}$ and exhibit them geometrically.

(i) Determine the limit points of the set $z_n = 2^n$ (n = 1, 2, ….).

(j) Write the function $f(z) = z^3 + z + 1$ in the form f (z) = u (x, y) + iv (x, y).

SECTION–B

Q2. Show that sum function of a uniformly convergent series of continuous functions is itself continuous.

Q3. Show that if $\{f_n(x)\}$ be a sequence of real valued continuous function defined on [a, b] such that $f_n \rightarrow f$ uniformly on [a, b] then $f \in [a, b]$ and

$$Lt \int_a^b f_n(x)\, dx = \int_a^b f(x)\, dx$$

Q4. Prove that the radius of convergence of the series obtained by integrating term by term has the radius of convergence as the original series.

Q5. Establish the identity :

$1 + z + z^2 + \ldots. + z^n = 1 - z^{n+1}/1-z$

Q6. Show that $f(z) = \bar{z}$ is nowhere differentiable.

Q7. Show that $u(x, y) = \sinh x \sin y$ is harmonic in some domain and find its harmonic conjugate $v(x, y)$

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.