Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages: 02**

Total No. of Questions: 09

B.Tech (Sem. – 1)

MATHEMATICS-I

Subject Code: BTAM-106-18

M Code: 75368

Date of Examination : 11-01-2023

Time: 3 Hrs. **Max. Marks: 60**

INSTRUCTIONS TO CANDIDATES:

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each, carrying EIGHT marks each.**
3. **Attempt any FIVE questions from SECTION B & C, selecting atleast TWO questions from each of these SECTIONS B & C.**

SECTION-A

1. Answer the following:

   a) Define a vector space.

   b) If A and B are square matrices. Is $AB = BA$? Justify.

   c) If A and B are symmetric matrices, then show that $AB - BA$ is skew-symmetric.

   d) Define eigenvalues of a matrix.

   e) Find the length of the Helix traced by

   $$r(t) = a\cos\ t\mathbf{i} + a\sin\ t\mathbf{j} + ct\mathbf{k}, \quad a > 0, 0 \le t \le 2\pi$$

   f) Find the unit normal vector to the surface $xy^2 + 2yz = 8$ at the point $(3, -2, 1)$.

   g) Define divergence of a vector field.

   h) Let f be a differentiable scalar field. Then calculate the value of $\nabla \times (\nabla f)$.

   i) Find the length of the arc given by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}, \quad 0 \le t \le \pi/2$.

   j) Evaluate $\int_C (x^2 - y^2)ds$, where C is the curve defined by $x = 3\cos\ t, y = 3\sin\ t, 0 \le t \le 2\pi$.

SECTION B

2. a) If x, y and z are different and

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & xz^2 & 1+z^3 \end{vmatrix} = 0$$

then show that $1 + \mathrm{xyz} = 0$

b) Solve the following system of equations using Gauss elimination method.

$$x - y + z = 1, \quad 2x + y - z = 2, \quad 5x - 2y + 2z = 5$$

3. a) Examine whether the following set of vectors are linearly independent.

$$(1,2,3,4), (2,0,1,2), (3,2,4,2)$$

b) Find the inverse of the matrix by using Gauss-Jordan method.

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 3 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

4. Find all the eigenvalues and the corresponding eigenvectors of the following matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

5. a) The eigenvalues of 3×3 matrix A corresponding to the eigenvalues 1,2,3 are $[-1,-1,1]^t, [0,1,0]^t, [0,-1,1]^t$ respectively. Find the matrix A

b) Prove that the eigenvectors of a symmetric matrix are real

SECTION C

6. a) Find directional derivative of the function $f(x,y) = x^2y^3 + xy$ at a point (2,1) in the direction of a unit vector that makes angle $\pi/3$ with x-axis.

b) If $\mathbf{a}$ is a constant and $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, show that curl $(\mathbf{a} \times \mathbf{r}) = 2\mathbf{a}$

7. a) Show that the vector field $\mathbf{v} = (y^2 - x^2 + y)\mathbf{i} + x(2y+1)\mathbf{j}$ is irrotational and find a scalar function $f(x,y,z)$ such that $\mathbf{v} = \text{grad}\, \boldsymbol{f}$

b) If $f(x,y) = x^2 - xy - y + y^2$, find all points where the directional derivative in the direction $\mathbf{b} = (\mathbf{i} + \sqrt{3}\mathbf{j})/2$ is zero.

8. a) Evaluate $\int_C (x+y)dx - x^2dy + (y+z)dz$ where C is $x^2 = 4y, z = x, 0 \le x \le 2$

b) Find the work done by the force $\mathbf{F} = -xy\mathbf{i} + y^2\mathbf{j} + z\mathbf{k}$ in moving a particle over a circular path $x^2 + y^2 = 4, z = 0$ from (2,0,0) to (0,2,0).

9. Verify Green's theorem for $f(x,y) = e^{-x}\sin y, g(x,y) = e^{-x}\cos y$ and C is the square with vertices at $(0,0), (\pi/2,0), (\pi/2,\pi/2), (0,\pi/2)$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.