Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐☐☐ Total No. of Pages : 02

Total No. of Questions : 18

B.Tech. (Automation & Robotics) (2018 Batch) (Sem.–3)

MATHEMATICS-III

Subject Code : BTAR-303-18
M.Code : 76502

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

Write briefly :

1. Find the Fourier series of the function $f(x) = |x|$ over the interval $[-2, 2]$.

2. Find Laplace transform of $e^{-t} \sin^2 t$.

3. State and prove Second Shifting Property for Laplace transform.

4. Find inverse Laplace transform of $\dfrac{2s-3}{s^2 + 4s + 13}$.

5. Express sum of Legendre polynomials $8P_4(x) + 2P_3(x) + P_0(x)$ in terms of powers of x.

6. For Legendre polynomial $P_n(x)$, show that $P_n(-x) = (-1)^n P_n(x)$

7. Form a partial differential equation by eliminating arbitrary function f from the relation
   $z = y^2 = 2f\left(\dfrac{1}{x} + \log y\right)$.

8. Solve $z(xp - yq) = y^2 - x^2$.

9. Show that the function $u(x, y) = 2x + y^3 - 3x^2y$ is harmonic.

10. State Cauchy Integral Theorem.

SECTION-B

11. Find the Fourier series expansion of the function

$$f(x) = \begin{cases} 0, & for -\pi \le x < 0 \\ 1, & for \ 0 \le x \le \pi \end{cases}. \text{ Deduce that } \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$$

12. State and prove Convolution Theorem for Laplace transform.

13. For Legendre polynomial $P_n(x)$, show that :

$$\int_{-1}^{1} P_m(x) P_n(x) dx = \frac{2}{2n+1}, \text{ for } m = n.$$

14. Solve by Charpit's method $z = p^2 x + q^2 y$.

15. Evaluate $\oint_C \frac{3z+5}{z^2 + 2z} dz$, $C : |z| = 1$.

SECTION-C

16. a) Using Laplace transform, solve $y'' - 6y' + 9y = e^{3t} t^2$, $y(0) = 2$, $y'(0) = 6$.

    b) Find inverse Laplace transform of $\dfrac{2s+1}{(s+2)^2(s-1)^2}$.

17. a) Solve Legendre differential equation $(1 - x^2) y'' - 2xy' + n(n+1)y = 0$.

    b) Using the method of separation of variables, solve

    $$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, \ u(x, 0) = x^2, \ u(0, t) = u(2\pi, t) = 0.$$

18. a) Find all Taylor and Laurent series expansions of $f(z) = \dfrac{1}{z^2+1}$ about the point $z = i$.

    b) Compute the residues at the singular points $z = 1, -2$ of

    $$f(z) = \frac{1 + z + z^2}{(z-1)^2 \ (z+2)}$$

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.