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Roll No. Total No. of Pages : 02

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B.Tech.(Automation & Robotics) (2011 & Onwards) (Sem.–3)

MATHEMATICS – III

Subject Code : BTAR-301

Paper ID : [A0130]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION–A

1. Write briefly :

(a) Find the Laplace transforms of $(t-1)^2 u\,(t-1)$

(b) Evaluate $\int_0^\infty \phi(t)\,\delta'(t-a)\,dt$.

(c) Show that $P_n(1) = 1$.

(d) Prove that $J_{-n}(x) = (-1)^n J_n(x)$.

(e) Show $\text{erf}(x) + \text{erf}_c(f) = 1$.

(f) Write necessary and sufficient conditions for $f(z)$ to be analytic.

(g) Show that the function $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic.

(h) Define conformal transformation.

(i) Define Cauchy's integral formula.

(j) Discuss singularity of $\frac{1}{\sin z - \cos z}$ at $z = \frac{\pi}{4}$.

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SECTION–B

2. Find the inverse Laplace transform of $f(p)=\dfrac{2p^2-1}{(p^2+1)(p^2+4)}$.

3. State and prove Rodrigue's formula.

4. If $U+V=\dfrac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$ and $f(z)$ = U + iV is an analytic function of Z = X + iY, Find $f(z)$ in the terms of Z.

5. Consider the map $W=\dfrac{1}{Z}$ and determine the region R in W-plane of the infinite strip R bounded by $\dfrac{1}{4}<y<\dfrac{1}{2}$.

6. Evaluate by using Cauchy-integral formula $\int_c \dfrac{z-1}{(z+1)^2(z-2)}dz$ where C is $|z-i|=2$.

SECTION–C

7. The Co-ordinates (x, y) of a particle moving along a plane curve at any time t are given by

$$\frac{dy}{dt}+2x=\sin 2t,\ \frac{dx}{dt}-2y=\cos 2t;\ (t>0)$$

It is given that at $t = 0$, $x = 1$ and $y = 0$. Show using transforms that the particle moves along the curve $4x^2 + 4xy + 5y^2 = 4$.

8. State and Prove/orthogonality of Bessel's function.

9. Using Contour integration, evaluate $\int_0^\infty \dfrac{dx}{1+x^2}$.

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