

**Roll No.**

**Total No. of Pages : 02**

**Total No. of Questions : 09**

**B.Tech.(IT) (2018 Batch) (Sem.-3)**

## MATHEMATICS-III

**Subject Code : BTAM-301-18**

**M.Code : 76393**

**Time : 3 Hrs.**

**Max. Marks : 60**

**INSTRUCTIONS TO CANDIDATES :**

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

## SECTION-A

- 1. Write briefly :**

- Show that the function  $f(x, y) = \frac{2x^2y}{x^4 + y^2}$  has no limit as  $(x, y)$  approaches  $(0, 0)$ .
- Find the local extreme values of the function  $f(x, y) = x^3 - y^3 - 2xy + 6$ .
- Sketch the region of integration for the integral

$$\int_0^{\sin x} \int_0^{\pi} y \, dy \, dx$$

and write an integral with the order of integration reversed.

- Define convergence of a series and give an example of a convergent series.
- Explain the limit comparison test.
- By inspection obtain the integrating factor and solve the differential equation :

$$x dx = y dy + 2 (x^2 + y^2) dx = 0$$

- g) Check whether the following differential equation exact.

$$(2x + e^y) dx + xe^y dy = 0$$

- h) Find the general solution of the differential equation  $y'' + 2y' + y = 0$

- i) Verify whether the linear combination of  $e^x$  and  $e^{-2x}$  is a solution of the differential equation

$$y'' + y' - 2y = 0$$

- j) Find the Wronskian of the functions  $x$ ,  $x^2$  and  $x^3$ .

### SECTION-B

2. Solve the following integral

$$\int_0^{\ln 2} \int_0^{\sqrt{(\ln 2)^2 - y^2}} e^{\sqrt{x^2 + y^2}} dx dy$$

by converting it into an equivalent polar integral.

3. For what values of  $x$  does the following power series converge ?

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$$

4. Solve the differential equation  $(3x^2y^3e^y + y^3 + y^2) dx + (x^3y^3e^y - xy) dy = 0$ .
5. Solve the differential equation  $y'' + 4y' + 4y = e^{-2x} \sin x$  by using method of variation of parameters.
6. Check the convergence of the following series

$$(i) \sum_{n=1}^{\infty} \frac{(2n)!}{n!n!} \quad (ii) \sum_{n=2}^{\infty} \frac{1}{\sqrt{n} \ln n}$$

### SECTION-C

7. a) Find the maximum and minimum values of the function  $f(x, y) = 3x + 4y$  on the circle  $x^2 + y^2 = 1$ .
- b) Find the volume in the first octant bounded by the coordinate planes and the surface  $z = 4 - x^2 - y$ .
8. State and prove Leibniz's test for alternating series.
9. Find the general solution of the equation  $x^3y''' - 3xy' + 3y = 16x + 9x^2 \ln x$ .

**NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.**