

Roll No. 

Total No. of Pages : 02

Total No. of Questions : 09

B.Tech.(ME) (2011 Onwards) (Sem.-5)

**MATHEMATICS-III**

Subject Code : BTAM-500

Paper ID : [A2127]

Time : 3 Hrs.

Max. Marks : 60

**INSTRUCTION TO CANDIDATES :**

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt ANY FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students have to attempt ANY TWO questions.

**SECTION-A**

1. Write briefly :

(a) Find the Fourier series of  $f(x) = \begin{cases} \frac{1}{2} + x, & -\frac{1}{2} < x < 0 \\ \frac{1}{2} - x, & 0 < x < \frac{1}{2} \end{cases}$ .

(b) Find Laplace transform of  $(1 + te^{-t})^3$ .

(c) Find inverse Laplace transform of  $\tan^{-1} \frac{2}{s}$ .

(d) Evaluate  $\int_0^1 \sqrt[3]{x \ln \left( \frac{1}{x} \right)} dx$ .

(e) Evaluate  $\int x^2 J_1(x) dx$ .

- (f) By eliminating arbitrary function, form a partial differential equation from

$$z = x^n f\left(\frac{y}{x}\right).$$

- (g) Solve the given partial differential equation  $p - q = \ln(x + y)$ .
- (h) Show that imaginary part of an analytic function is harmonic.
- (i) Find the orthogonal trajectories of the family of curves  $x^3 y - x y^3 = c = \text{constant}$ .
- (j) State Cauchy's integral formula.

### SECTION-B

2. Find the Fourier series of  $f(x) = |\cos x|$  in the interval  $(-\pi, \pi)$ .
3. Using Laplace transform, solve  $y'' + 4y = u(t - 2)$ ,  $y(0) = 0$ ,  $y'(0) = 1$ , where  $u(t)$  is a unit step function.
4. Using Frobenius method, find the general solution of  
$$8x^2 y'' + 10xy' - (1 + x)y = 0.$$
5. Solve given partial differential equation  $(2D_x^2 + 5D_x D_y + 2D_y^2)z = 0$ .
6. Verify that  $u = 3xy^2 - x^3$  is harmonic and find its conjugate harmonic function.

### SECTION-C

7. (a) Use Laplace Transform to solve given system of simultaneous differential equations  
$$\frac{dx}{dt} - y = e^t, \frac{dy}{dt} + x = \sin t, \text{ where } x(0) = 1 \text{ and } y(0) = 0.$$
- (b) For Legendre polynomials  $P_n(x)$  show that 
$$\int_{-1}^1 P_m(x) P_n(x) dx = \frac{2}{2n+1}$$
when  $m = n$ .
8. (a) State and prove Convolution for Laplace transform.  
(b) A bar of 30 cm length has its ends kept at  $20^\circ$  and  $80^\circ$  respectively until steady-state condition prevail. The temperature at each end is then suddenly reduced to  $0^\circ$  and maintained thereafter, Find the temperature in bar.
9. (a) Find Laurent series of  $\frac{z}{(1+z)(z+2)}$  about  $z_0 = -2$ .  
(b) Using Residue theorem, evaluate 
$$\int_C \frac{\tan z dz}{(z^2 - 1)}, C: |z| = \frac{3}{2}.$$