Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 11

M.Sc. (Physics) (2018 Batch) (Sem.–1)

MATHEMATICAL PHYSICS–I

Subject Code : MSPH-411-18

M.Code : 75122

Time : 3 Hrs. **Max. Marks : 70**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains SEVEN questions carrying FIVE marks each and students have to attempt any SIX questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. Answer briefly :

a) Find $\Gamma\left(-\frac{5}{2}\right)$.

b) Solve the integral $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}}$.

c) Define Dirac delta function in one dimension. Also state **any four** properties.

d) Define isolated singularity with suitable example.

e) Find whether $f(z) = |z|^2$ is an analytic function or not?

f) A coin is tossed three times. What is the probability of atleast two tails in succession?

g) Write down the solution of the differential equation $x^2y'' + xy' + (x^2 - 1/9)y = 0$.

h) Discuss where Hermite and Legendre polynomials are used in physics?

i) State Dirichlet and Neumann boundary conditions.

j) Find standard deviation of the function $f(x) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{x^2}{8}}$

SECTION-B

2. Derive duplication formula for gamma function.

$$\Gamma(2n)=\frac{1}{\sqrt{\pi}}2^{2n-1}\Gamma(n)\Gamma\left(n+\frac{1}{2}\right)$$

3. Prove the orthonormality condition of Legendre polynomials.

4. a) Show that $\sqrt{\pi x/2}J_{1/2}(x)=\sin x$.

   b) Show that $\lim_{x\to 0} J_1(x)/x=\frac{1}{2}$.

5. Given an analytic function $f(x, y) = \phi(x, y) + i\,\psi(x, y)$, where $\phi(x, y) = x^2 + 4x - y^2 + 2y$. Find $\psi(x, y)$.

6. Find Laurent series of the function $f(z) = \frac{1}{(z-1)(z-2)}$ about $z = 0$ for the region a) $|z| < 1$ and b) $1 < |z| < 2$.

7. Discuss Binomial, Normal and Poissonian distributions.

8. A uniform bar of length l is heated so that its both ends are at 0 temperature. If initially the temperature is given as $f(x) = cx(l - x)/l^2$, where c is a constant. Find the temperature of different points at time t.

SECTION-C

9. State and prove Cauchy residue theorem and use it to evaluate $\int_0^{2\pi}\frac{d\theta}{(5-3\sin\theta)^2}$.

10. a) Prove Rodrigues' formula $P_l(x)=\frac{1}{2^l\,l!}\frac{d^l}{dx^l}(x^2-1)^l$.

    b) Express the polynomial $3x^2 + x - 1$ as a linear combination of Legendre polynomials.

11. a) Solve the integral $\int_0^1(\ln x)^{1/3}dx$.

    b) Solve the integral $\int_0^3 x^2\delta(x+2)\,dx$.

    c) For the following function, locate and classify the singularities in finite z plane.

    (i) $f(z)=\frac{1}{\sin\frac{\pi}{z}}$. (ii) $f(z)=\frac{\sin z}{z^4}$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.