Roll No. | | | | | | | | | | | | Total No. of Pages : 02

Total No. of Questions : 07

M.Sc Mathematics (2017 Batch) (Sem.–1)

MATHEMATICAL METHODS

Subject Code : MSM-105

M.Code : 74724

Time : 3 Hrs. **Max. Marks : 80**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.**
2. **SECTION - B & C have THREE questions in each section carrying SIXTEEN marks each.**
3. **Select atleast TWO questions from SECTION - B & C EACH.**

SECTION-A

1. **Answer briefly :** (2×8=16)

   a) Find the Fourier transform of $e^{-a|t+1|}$, $a > 0$.

   b) Define eigen values of a kernel in the integral equation.

   c) Find the Hankel transform of e^{-x}.

   d) Write change of scale property for Laplace transform.

   e) Determine Z $\{\sin(n+1)\theta\}$.

   f) State first shifting property of Laplace transform.

   g) State convolution theorem for Laplace transforms.

   h) Give an example of Volterra and Fredholm integral equation.

SECTION-B

2. a) Find the Laplace transform of the function $f(t) = t^{5/2}$ given that $\Gamma(1/2) = \sqrt{\pi}$. (8)

   b) Solve the difference equation $y_{n+2} + 5y_{n+1} + 4y_n = 2^n$, $y_0 = 1$, $y_1 = -4$ using Z-transform. (8)

3. a) Find the geodesics on a right circular cylinder of radius a. (8)

b) Find the inverse Laplace of the function $f(s) = \frac{16+3s}{s^2-8s+20}$. (8)

4. a) Solve the initial value problem $y'' - 5y' + 4y = e^{2t}$, $y(0) = 19/12$ and $y'(0) = 8/3$. (6)

b) The temperature distribution $u(x, t)$ in a thin, homogeneous, infinite bar can be modeled by the initial value problem $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, $-\infty < x < \infty$, $t > 0$, $u(x, 0) = f(x)$, $u(x, t)$ is finite as $x \to \pm\infty$. (10)

SECTION-C

5. a) Find the inverse Z transform of F (z), where $F(z) = \frac{7z-11z^2}{(z-1)(z-2)(z+3)}$. (8)

b) Using convolution, find $F^{-1}\left[\frac{1}{12+7iw-w^2}\right]$ (8)

6. a) Solve the boundary value problem $y'' - y + x = 0$ $(0 \le x \le 1)$, $y(0) = y(1) = 0$ by Rayleigh-Ritz method. (8)

b) Solve $y(x) = x + 2\int_0^x \cos(x-t)\, y(t)$. (8)

7. a) Find the curves on which the functional $\int_0^1 (y'^2 + 12xy)\, dx$ with $y(0) = 0$ and $y(1) = 1$.(8)

b) Determine the eigen values and eigen functions for the following integral equation with degenerate kernels: $y(x) = \lambda \int_0^{\pi/4} (3x-2)\, t\, y(t)\, dt$. (8)

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.