Roll No.

Total No. of Pages : 02

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M.Sc Mathematics (2017 Batch) (Sem.–2)

PARTIAL DIFFERENTIAL EQUATIONS

Subject Code : MSM-204

Paper ID : [75011]

Time : 3 Hrs. **Max. Marks : 80**

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.
2. SECTION - B & C. have THREE questions in each section carrying SIXTEEN marks each.
3. Select atleast TWO questions from SECTION - B & C EACH.

SECTION-A

1. Answer the following :

a) Form a partial differential equation by eliminating a, b from $z = (x + a)(y + b)$.

b) Solve $p + q = z/a$.

c) Solve $p^2 + q^2 = 1$.

d) Find the complete integral of $z = px + qy + p^2 + q^2$.

e) Solve $r = a^2 t$.

f) Find particular integral of $(D^3 - 10D^2D' + D'^3)\, z = \cos(2x + 3y)$.

g) State Laplace equation and diffusion equation.

h) Classify the following equation as elliptic, parabolic or hyperbolic $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{\partial z}{\partial y}$.

SECTION-B

2. a) Find the surface whose tangent planes cut off an intercept of constant length k from the axis of z.

   b) Solve $(p^2 + q^2)\, y = qz$ using Charpit method.

3. Solve $(x_2 + x_3)\,(p_2 + p_3)^2 + z\, p_1 = 0$ by using Jacobi's Method.

4. a) Find equation of surface which cuts surfaces of the system $z\,(x + y) = \lambda\,(3z + 1)$ orthogonally and which passes through the curve $x^2 + y^2 = 1$, $z = 1$.

   b) Find the general solution of $(D_x^2 - \alpha^2 D_y^2)\, z = x^2$.

SECTION-C

5. a) The faces $x = 0$ and $x = 1$ of infinite slab are maintained at zero temperature and $u\,(x, t) = f(x)$ at $t = 0$. Determine the temperature at a subsequent time t.

   b) Find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection $f(x) = k\,(\sin x - \sin 2x)$.

6. Derive Heat Diffusion Equation and obtain the solution using method of separation of variables.

7. a) Solve $\dfrac{\partial^2 z}{\partial x^2} - 2\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 0$ by the method of separation of variables.

   b) Solve $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$, subject to $u\,(x, 0) = u\,(x, m) = 0$ where $0 \le x \le \ell$ and $u\,(0, y) = 0$, $u\,(\ell, y) = F\,(y)$ where $0 \le y \le m$.