Roll No. $\square$ Total No. of Pages : 02
Total No. of Questions: 07
M.Sc Mathematics (2017 Batch) (Sem.-2)

PARTIAL DIFFERENTIAL EQUATIONS
Subject Code : MSM-204
M.Code : 75011

Time : 3 Hrs.
Max. Marks : 80

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.
2. SECTION - B \& C. have THREE questions in each section carrying SIXTEEN marks each.
3. Select atleast TWO questions from SECTION - B \& C each.

## SECTION-A

1. Answer the following :
a) Form the partial differential equation by eliminating the arbitrary function from $f\left(x^{2}+y^{2}, z-x y\right)=0$.
b) Classify the partial differential equation $u_{x x}+u_{y y}=f(x, y)$.
c) Find the complete solution of the partial differential equation $p^{3}-q^{3}=0$
d) Write all possible solutions of two-dimensional Laplace equation.
e) Solve $\left(D^{3}-D^{2} D^{\prime}-8 D D^{\prime 2}+12 D^{\prime 3}\right) z=0$.
f) Give an example of a linear and nonlinear partial differential equation.
g) Find the partial differential equation of all surfaces of revolution having $z$-axis as the axis of rotation.
h) State the assumptions in deriving one-dimensional wave equation.

## SECTION-B

2. a) Find the partial differential equation of all planes which are at a constant distance $k$ units from the origin.
b) Find the equation of the system of surfaces which cut orthogonally the cones of the system $x^{2}+y^{2}+z^{2}=c x y$.
3. a) Show that the characteristics of the equation $R r+S s+T t=f(x, y, z, p, q)$ are invariant with respect to any transformations of the independent variables.
b) Find the complete integral of the partial differential equation $\left(p^{2}+q^{2}\right) x=p z$ and deduce the solution which passes through the curve $x=0, z^{2}=4 y$.
4. a) Solve the equation $r+(a+b) s+a b t=x y$ by Monge's method.
b) Solve : $\left(\mathrm{D}^{3}-2 \mathrm{D}^{2} \mathrm{D}^{\prime}\right) z=2 e^{2 x}+3 x^{2} y$.

## SECTION-C

5. a) Solve $p^{2} x+q^{2} y=z$ by Jacobi's method.
b) Find the complete solution of $p^{2}+x^{2} y^{2} q^{2}=x^{2} z^{2}$.
6. Find the solution of three dimensional diffusion equation in the region $0<x<a$, $0<y<b, 0<z<c, t>0, \frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}+\frac{\partial^{2} u}{\partial z^{2}}=\frac{1}{k} \frac{\partial u}{\partial t}$ with the boundary and initial conditions: $u(0, y, z, t)=0=u(a, y, z, t) ; u(x, 0, z, t)=0=u(x, b, z, t) ; u(x, y, 0, t)=0=$ $u(x, y, c, t)$ and $u(x, y, z, 0)=f(x, y, z)$.
7. A tightly stretched violin string of length $l$ and fixed at both ends is plucked at $x=l / 3$ and assumes initially the shape of a triangle of height $a$. Find the displacement $u(x, t)$ at any distance $x$ and any time $t$ after the string is released from rest.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.

