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M.Sc Mathematics (2017 Batch) (Sem.–3)

FUNCTIONAL ANALYSIS

Subject Code : MSM-304

M.Code : 75384

Time : 3 Hrs. **Max. Marks : 80**

INSTRUCTION TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.**
2. **SECTION - B & C. have THREE questions in each section carrying SIXTEEN marks each.**
3. **Select atleast TWO questions from SECTION - B & C EACH.**

SECTION-A

1. **Write briefly :**

   (a) It is always possible to define a Norm on any vector space?

   (b) State and prove Schwarz inequality.

   (c) Prove that if N is a normal operator on H, then $\|N^2\| = \|N\|^2$.

   (d) State and prove Parallelogram Law.

   (e) Prove that an operator T on a Hilbert space H can be written as $T = T_1 + iT_2$, where T_1 and T_2 are self adjoint operators on H.

   (f) If N_1, N_2 are normal operators on H, then what can you say about $N_1 + N_2$.

   (g) Give one example of standard orthogonal basis for Hilbert space L_2 associated with the measure space $[0, 2\pi]$. Also convert orthogonal basis to orthonormal basis.

   (h) Derive Parseval's equation for a given complete orthonormal set $\{e_i\}$ in a Hilbert space H.

SECTION-B

2. (a) Give example to justify the below given statement, where notation used below have standard meaning

$$C_{00} \subset C\ \ell^p \subset c_0 \subset c \subset \ell^\infty.$$

(b) State and prove Uniform Boundedness principle.

3. (a) Show that the $B(N)$, the set of linear operators on Normed linear space, is an algebra.

(b) Let B be a Banach space and N a normed linear space. If $\{T_i\}$ is a non-empty set of continuous transformations of B into N with the property $\{T_i(x)\}$ is a bounded subset of N for each vector x in B, then prove that $\{||T_i||\}$ is a bounded set of numbers

4. State and prove Closed graph Theorem.

SECTION-C

5. a) Let E be an orthonomal set in an inner product space X. Then E is an orthonormal basis iff $E^\perp = \{0\}$.

(b) Let X be a Hilbert space and $A \in B(X)$, then A is unitary if and only is A is surjective and $||Ax|| = ||x||$ for every $x \in X$.

6. (a) Let M be a closed linear subspace of a Normed linear space N, and let x be a vetor not in M. If d is the distance from x to M, show that there exist a functional f in N such that $f\{M) = 0, f\{x) = 1$, and $||f|| = l/d$.

(b) Prove that every Hilbert space H is finite dimensional iff every complete orthonormal in H is a basis.

7. Prove that every Hilbert space H is reflexive.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.