M.Sc. (IT) / MCA   (Sem.–3)

DISCRETE STRUCTURES & OPTIMIZATION
Subject Code : PGCA-1917
Paper ID : 78393

Time : 3 Hrs.   **Max. Marks : 70**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying TEN marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

Answer briefly :

1. i)   Is the set A = $\{x : x^2 – 7x + 12 = 0$ and $x^2 – 4x + 3 = 0\}$ empty? Justify.

    ii) In how many ways 3 girls and 5 boys can sit at a round table so that no two girls can sit together.

2. i)   State and prove De Morgan's Laws for sets.

    ii) If A and B are two subsets of a universal set U with $n(U) = 500$, $n(A) = 100$, $n(B) = 200$ and $n(A \cap B) = 50$, find $n(A' \cap B')$.

3. i)   Solve $a_n = 6a_{n-1} – 8a_{n-2}$, $a_0 = 4$, $a_1 = 10$.

    ii) State Exclusion-Inclusion Principle.

4. Find x and y if $(x + 2, 4) = (5, 2x + y)$.

5. Let A and B be sets with cardinalities m and n respectively. Find the number of one-to-one mapping from A to B where $m < n$.

6. What will be the chromatic number of complete graph with 'n' vertices? Explain.

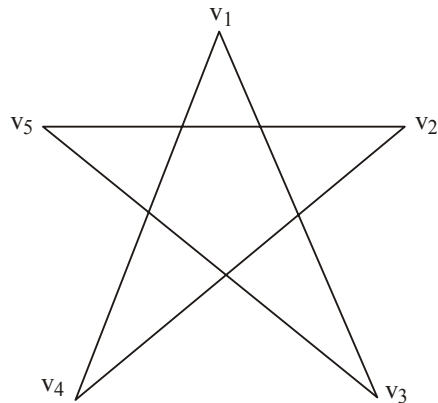7. Draw the complement of graph given in fig. below :

FIG. 1

8. Let (G, o) be a group. Show that if (G, o) is an Abelian group, then show that $(a \ o \ b)^2 = a^2 o b^2$ for all a and b in G.

9. Show that Z, the set of integers, is an integral domain.

10. If $(B, +, .., ')$ is a Boolean algebra, then show that $a + a = a \ \forall \ a \in B$.

SECTION-B

11. i) Prove that a set containing n distinct elements has 2^n subsets.

    ii) If A, B, C be any three sets, then show that $A - (B \cap C) = (A - B) \cup (A - C)$.

12. i) In a joint family of 12 persons, 7 take tea, 6 take milk and 2 take neither. How many members take both tea and milk?

    ii) Solve $S(k) - 4S(k-1) + 3S(k-2) = k^2$.

13. i) Prove that the chromatic number of complete bipartite graph $k_{m, n}$. where m and n are +ve integers, is two.

    ii) Can a graph with seven vertices be isomorphic to its complement? Justify.

14. i) If H and K are two subgroups of G, then prove that $H \cap K$ is also a subgroup of G.

    ii) Consider a Boolean expression $f(a, b, c) = ((ab)' \ c)' \ ((a' + c) \ (b' + c'))'$ into a sum-of-products form.

SECTION-C

15. i) Prove that A × (B – C) = (A × B) – (A × C).

    ii) Each student in a class of 50, studies atleast one of the subject English, Maths and Physics. 36 study English, 32 Physics and 26 Maths. 5 study English and Physics. 14 Maths and Physics and 2 English, Maths and Physics. Find the number of students who study (i) English and Maths (ii) English, Maths but not Physics.

16. i) Solve $S(k) - 7S(k - 2) + 6S(k - 3) = 0$, $S(0) = 8$, $S(1) = 6$, $S(2) = 22$.

    ii) Consider the function $f : N \times N \to R$ define by $f(x, y) = (2x + 1) 2^y - 1$, where N is the set of natural numbers including zero. Show that f is bijective.

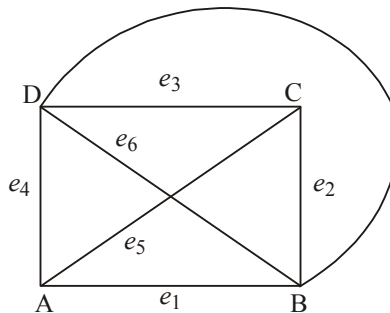17. i) Does the graph given below possess an Euler's circuit ?

FIG. 2

    ii) Find the chromatic number of the complete graph k_6, k_8 and k_n.

18. i) "*Determine whether a semi-group with more than one idempotent element can be a group*". Justify.

    ii) Prove that an element a in Z_n is a unit iff a and n are relatively prime.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.